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A numerical proximity between Earth's dimensionless surface potential and a Planck–horizon scale ratio

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ABSTRACT

I report a numerical proximity between Earth's dimensionless surface potential $\Phi_\oplus \equiv GM_\oplus/(c^2 R_\oplus)$ and a dimensionless scale ratio constructed from three length scales: Earth's radius $R_\oplus$, a present-epoch cosmic horizon scale $R_{\rm obs}$, and a mesoscopic length $L_{\rm GM}$ defined as the geometric mean between the Planck length $\ell_{\rm P}$ and the horizon diameter $D_{\rm obs} \equiv 2R_{\rm obs}$. The ratio $\chi \equiv R_\oplus^2/(L_{\rm GM} R_{\rm obs})$ evaluates to $\chi \approx 7.61 \times 10^{-10}$, compared with $\Phi_\oplus \approx 6.96 \times 10^{-10}$, a $\sim 9.4\%$ discrepancy. I provide a transparent numerical evaluation and document sensitivity to standard definitional choices. No physical mechanism is proposed.

INTRODUCTION

Dimensionless ratios are routinely used to characterize the "strength" of gravity. A common example is the dimensionless surface potential of a gravitating body, $\Phi \equiv GM/(c^2 R)$. In this note, I record a numerical proximity between Earth's surface potential $\Phi_\oplus$ and a dimensionless scale ratio built from three length scales: (i) Earth's radius $R_\oplus$, (ii) a cosmic horizon length $R_{\rm obs}$, and (iii) a mesoscopic length $L_{\rm GM}$ defined as the geometric mean between the Planck length $\ell_{\rm P}$ and the cosmic horizon diameter $D_{\rm obs}$:

$$L_{\rm GM} \equiv \sqrt{\ell_{\rm P} D_{\rm obs}}, \quad D_{\rm obs} = 2R_{\rm obs}. \tag{1}$$

The dimensionless quantity of interest is:

$$\chi \equiv \frac{R_\oplus^2}{L_{\rm GM} R_{\rm obs}}. \tag{2}$$

EVALUATION

I use the exact speed of light $c \equiv 299\,792\,458\,\mathrm{m\,s^{-1}}$ and the Planck length $\ell_{\rm P} \equiv 1.616 \times 10^{-35}\,\mathrm{m}$ (Mohr et al. 2025). For the cosmic scale, I adopt the present-day particle horizon $R_{\rm obs} \approx 47\,\mathrm{Gly}$ ($\approx 4.45 \times 10^{26}\,\mathrm{m}$) (NASA 2017). For Earth, I use the mean radius $R_\oplus \equiv 6.371 \times 10^6\,\mathrm{m}$ and geocentric gravitational parameter $\mu_\oplus \equiv 3.986 \times 10^{14}\,\mathrm{m^3\,s^{-2}}$ (Petit & Luzum 2010).

Using these baseline values, the geometric-mean length is $L_{\rm GM} \approx 1.20 \times 10^{-4}\,\mathrm{m}$. The scale ratio evaluates to:

$$\chi = \frac{(6.371 \times 10^6\,\mathrm{m})^2}{(1.199 \times 10^{-4}\,\mathrm{m})(4.447 \times 10^{26}\,\mathrm{m})} \approx 7.61 \times 10^{-10}. \tag{3}$$

For comparison, Earth's dimensionless surface potential is:

$$\Phi_\oplus = \frac{\mu_\oplus}{c^2 R_\oplus} \approx 6.96 \times 10^{-10}. \tag{4}$$

The fractional discrepancy is $\Delta = (\chi - \Phi_\oplus)/\Phi_\oplus \approx 9.4\%$.

SENSITIVITY

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Table 1. Sensitivity of χ to the choice of cosmic scale ($R_\oplus$ fixed).

Cosmic Scale	Value	χ	Δ
Particle Horizon (Baseline)	47 Gly	7.61×10^{-10}	+9.4%
Particle Horizon (Rounded)	46.5 Gly	7.74×10^{-10}	+11.1%
Hubble Length (c/H_0)	14.5 Gly	4.44×10^{-9}	+538%

The proximity relies on identifying the "observable universe" scale with the particle horizon. Table 1 demonstrates that using the Hubble length (c/H_0) instead breaks the relationship. Variations in Earth radius conventions (e.g., WGS-84 polar vs. equatorial) affect the result by $< 1\%$.

This note reports the geometric proximity only and does not propose a physical mechanism. The specific reproduction script (`calc_proximity.py`) is available within the project repository (Nimmo 2025).

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